

# EE 230

## Lecture 42

### Data Converters

#### Nonideal Effects

## Review from Last Time:

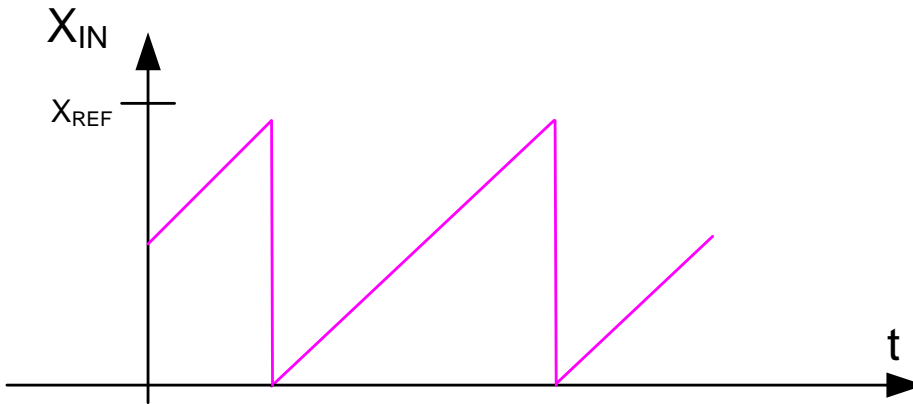
# Characterization of Quantization Noise

## Saw tooth excitation

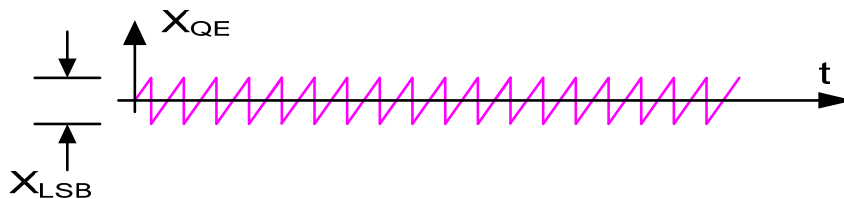
$$X_{QRMS} = \frac{X_{LSB}}{\sqrt{12}}$$

$$SNR = \frac{X_{SIG-RMS}}{X_{NOISE-RMS}}$$

What is  $X_{SIG-RMS}$  ?



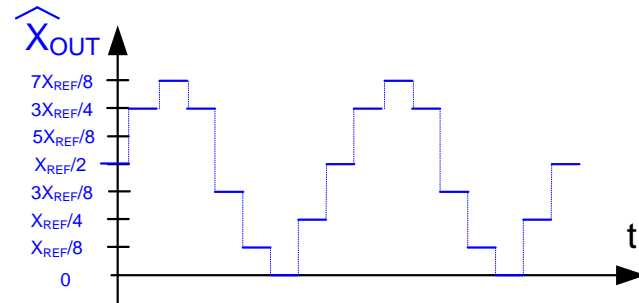
$$X_{SIG-RMS} = \frac{X_{REF}}{\sqrt{12}}$$



# Characterization of Quantization Noise

## Sinusoidal excitation

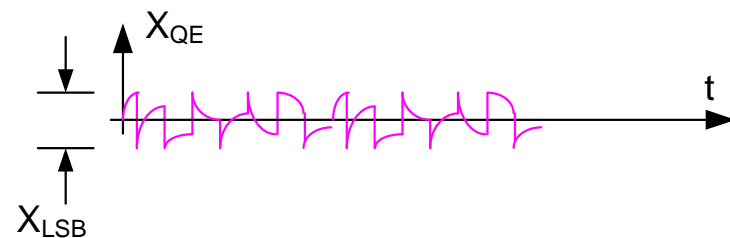
- Consider an ADC



Quantization noise is difficult to analytically characterize

Still need RMS value of  $X_{QE}(t)$

Will consider error in interpreted output

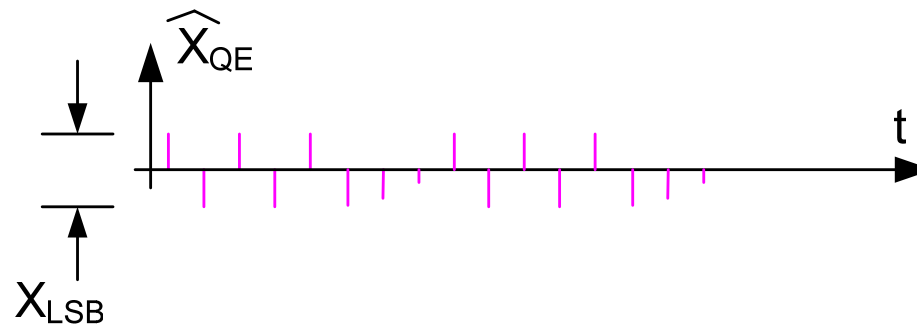
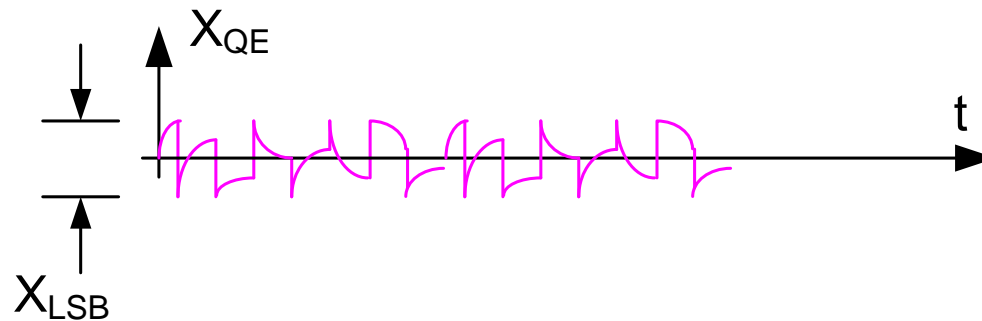


# Characterization of Quantization Noise

## Sinusoidal excitation

- Consider an ADC

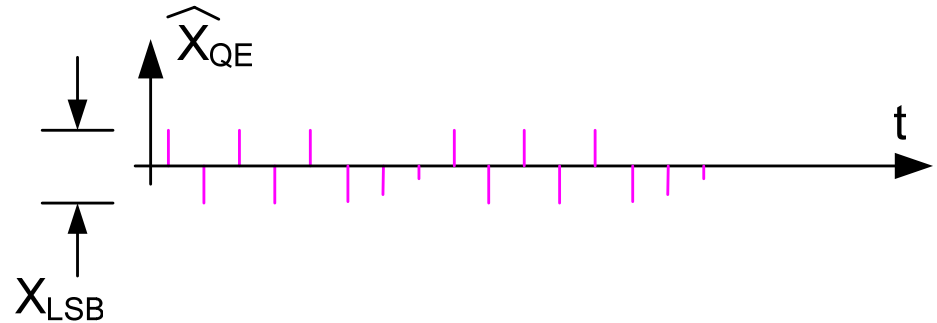
Will consider error in interpreted output



# Characterization of Quantization Noise

## Sinusoidal excitation

- Consider an ADC (clocked)



Theorem: If  $n(t)$  is a random process, then  $V_{\text{RMS}} \cong \sqrt{\sigma^2 + \mu^2}$

provided that the RMS value is measured over a large interval where the parameters  $\sigma$  and  $\mu$  are the standard deviation and the mean of  $\langle n(kT) \rangle$

This theorem can thus be represented as

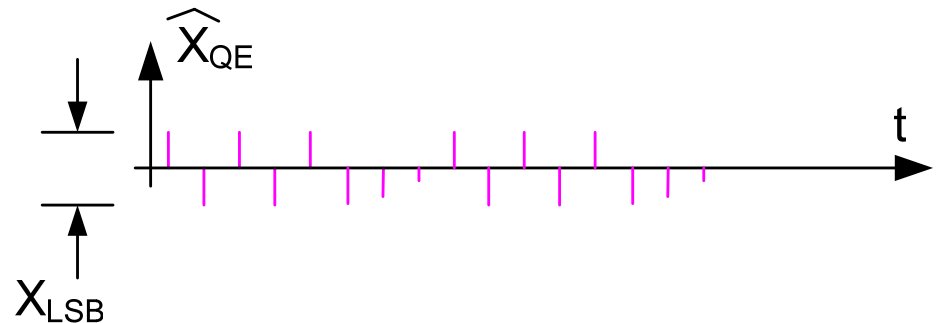
$$V_{\text{RMS}} \cong \sqrt{\frac{1}{T_L} \int_{t_1}^{t_1+T_L} n^2(t) dt} \cong \sqrt{\sigma^2 + \mu^2}$$

where  $T$  is the sampling interval and  $T_L$  is a large interval

# Characterization of Quantization Noise

## Sinusoidal excitation

- Consider an ADC



The quantization noise samples of the ADC output are approximately uniformly distributed between in the interval  $[-X_{\text{LSB}}/2, X_{\text{LSB}}/2]$

$$\langle n(kT) \rangle \sim U[-X_{\text{LSB}}/2, X_{\text{LSB}}/2]$$

A random variable that is  $U[a,b]$  has distribution parameters  $\mu$  and  $\sigma$  given by

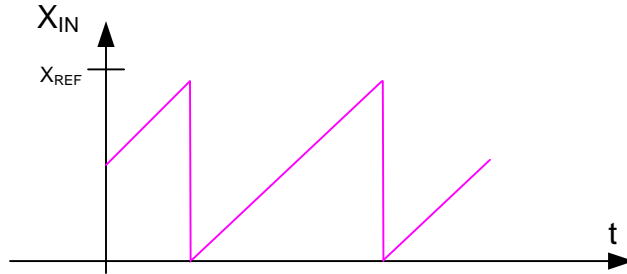
$$\mu = \frac{A + B}{2} \quad \sigma = \frac{B - A}{\sqrt{12}}$$

thus, the random variable  $n(kT)$  has distribution parameters

$$\mu = 0 \quad \sigma = \frac{X_{\text{LSB}}}{\sqrt{12}}$$

# Characterization of Quantization Noise

## Saw tooth excitation



$$X_{Q-RMS} \cong \frac{X_{LSB}}{\sqrt{12}}$$

$$SNR = 2^n$$

$$SNR_{dB} = 6.02n$$

## Sinusoidal excitation



$$SNR = 1.225 \cdot 2^n$$

$$SNR_{dB} = 6.02n + 1.76$$

Although derived for an ADC, same expressions apply for DAC

SNR for saw tooth and for triangle excitations are the same

SNR for sinusoidal excitation larger than that for saw tooth by 1.76dB

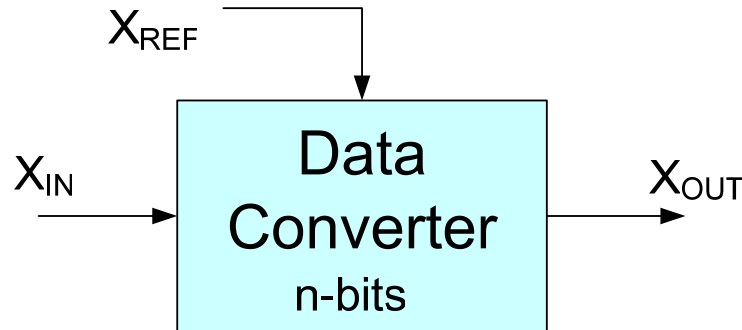
SNR will decrease if input is not full-scale

Equivalent Number of Bits (ENOB) often given relative to quantization noise  $SNR_{dB}$

Remember – quantization noise is inherent in an ideal data converter!

Review from Last Time:

# Equivalent Number of Bits (ENOB)



$$\text{ENOB} = \frac{\text{SNR} - 1.76}{6.02}$$

$$\text{ENOB} = \frac{\text{SNDR} - 1.76}{6.02}$$

These definitions of ENOB are based upon noise or noise and distortion

Some other definitions of ENOB are used as well – e.g. if one is only interested in distortion, an ENOB based upon distortion can be defined.

ENOB is useful for determining whether the number of bits really being specified is really useful



# Engineering Issues for Using Data Converters

## 1. Inherent with Data Conversion Process

- Amplitude Quantization
  - Time Quantization
- (Present even with Ideal Data Converters)

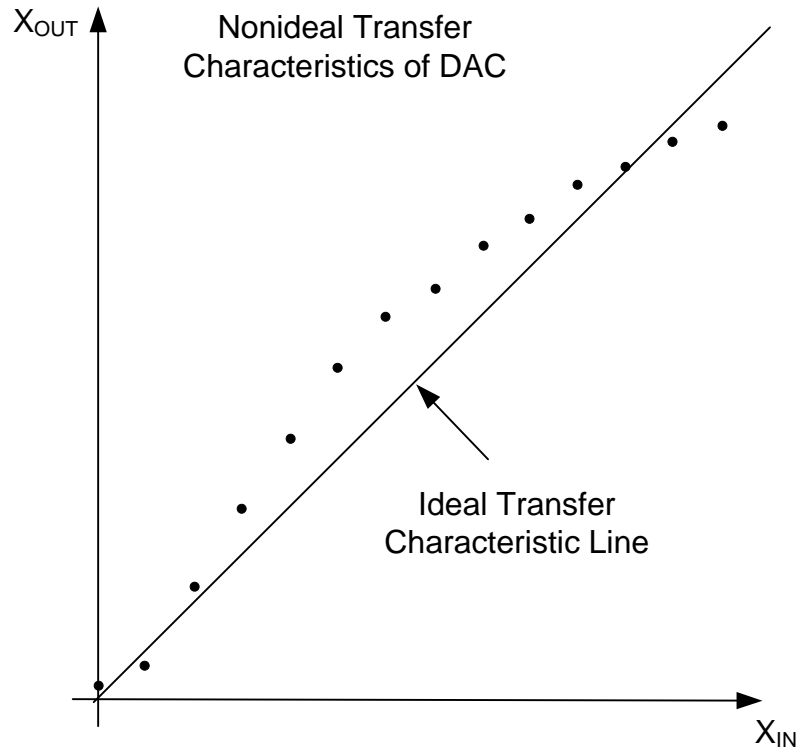
## → 2. Nonideal Components

- Uneven steps
  - Offsets
  - Gain errors
  - Response Time
  - Noise
- (Present to some degree in all physical Data Converters)

How do these issues ultimately impact performance ?

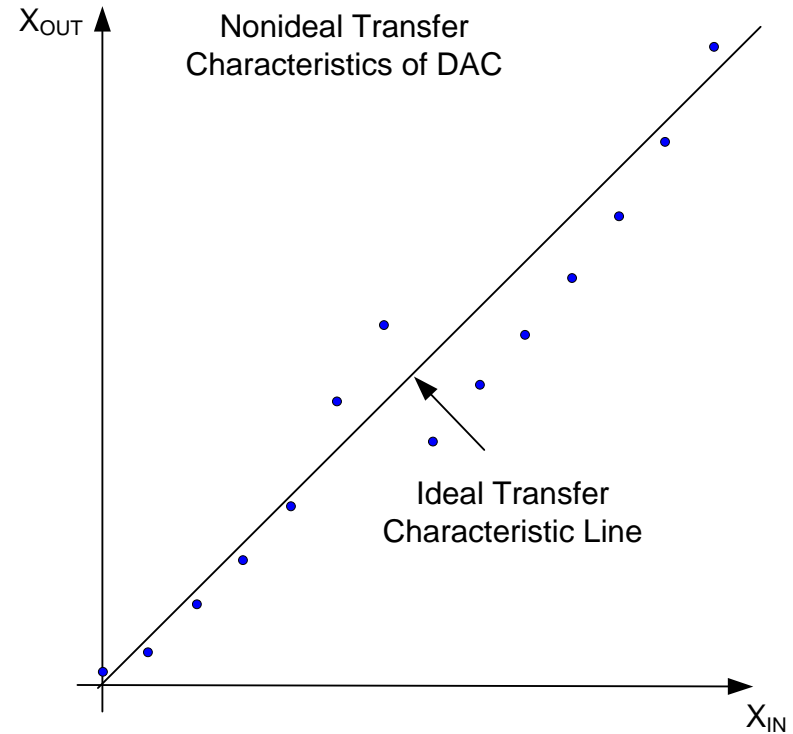
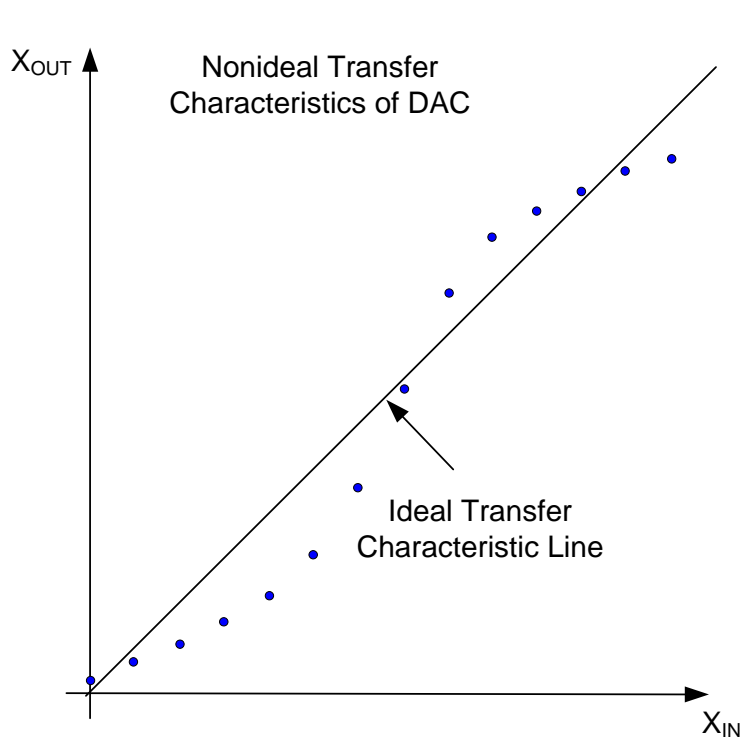
# Nonideal Transfer Characteristics

Uneven Steps



# Nonideal Transfer Characteristics

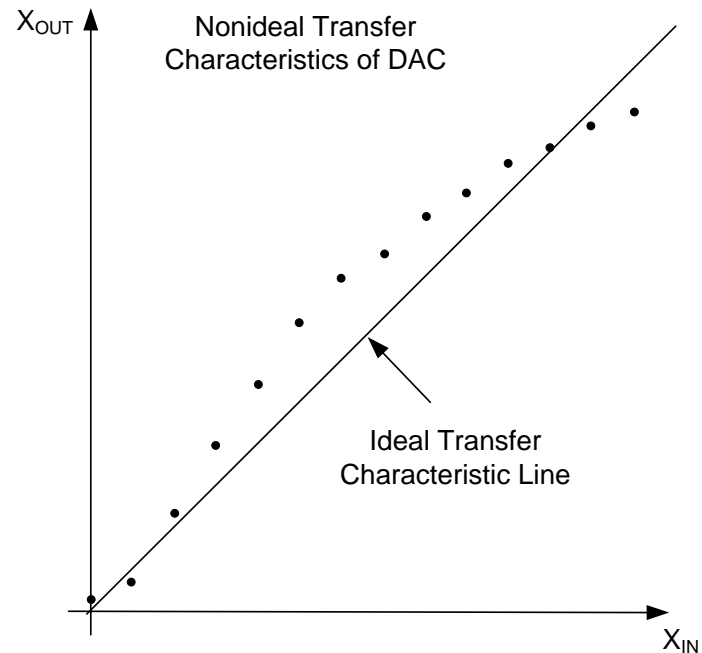
## Uneven Steps



Actual transfer characteristics can vary considerably from one device to another

# Nonideal Transfer Characteristics

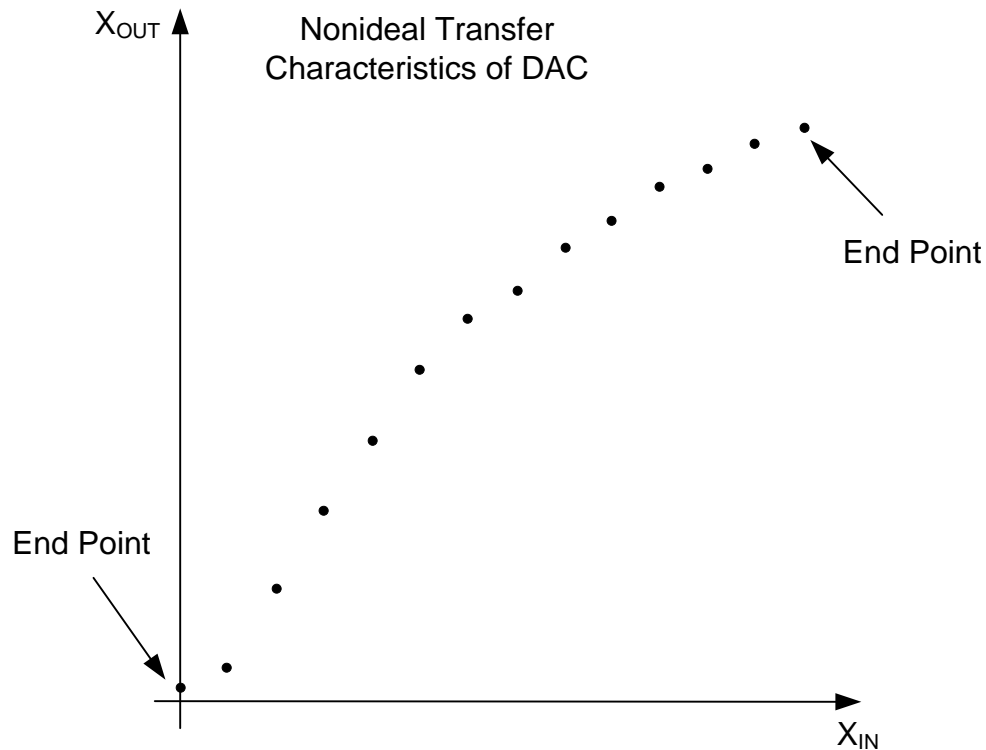
## Uneven Steps



This is termed a nonlinearity in the data converter

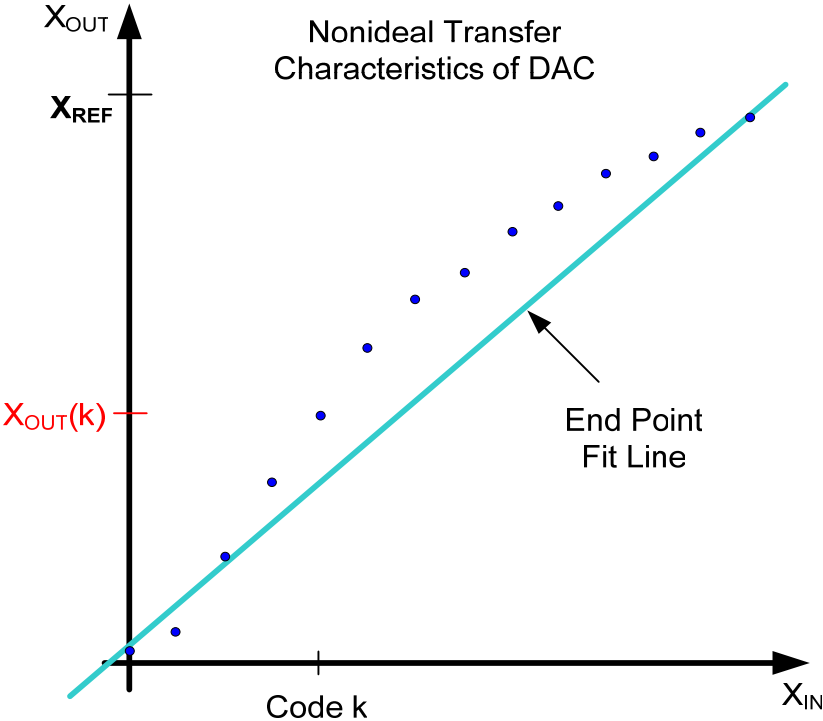
Linearity metrics (specifications) include INL, DNL, THD and SFDR

# Characterization of Nonlinearities



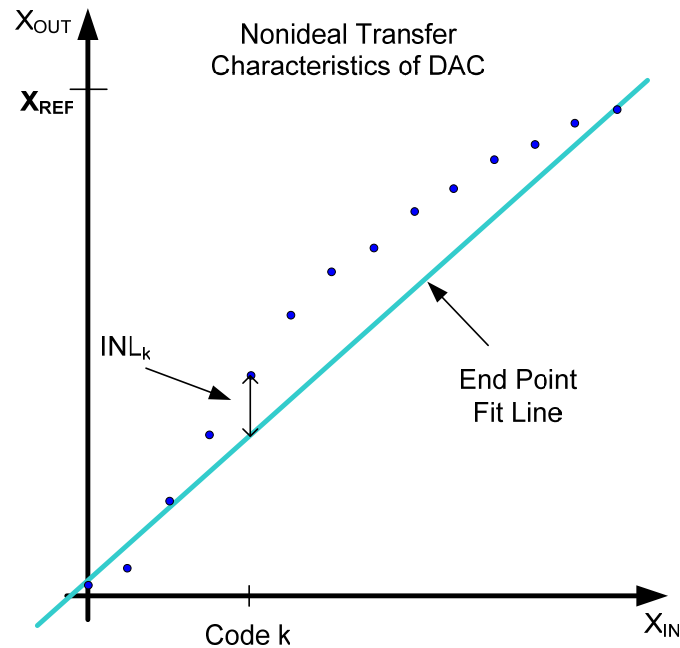
End points are the outputs at the two extreme Boolean inputs

# Characterization of Nonlinearities



End point fit line

# Characterization of Nonlinearities



Linearity metrics:  
→ INL  
DNL  
THD  
SFDR

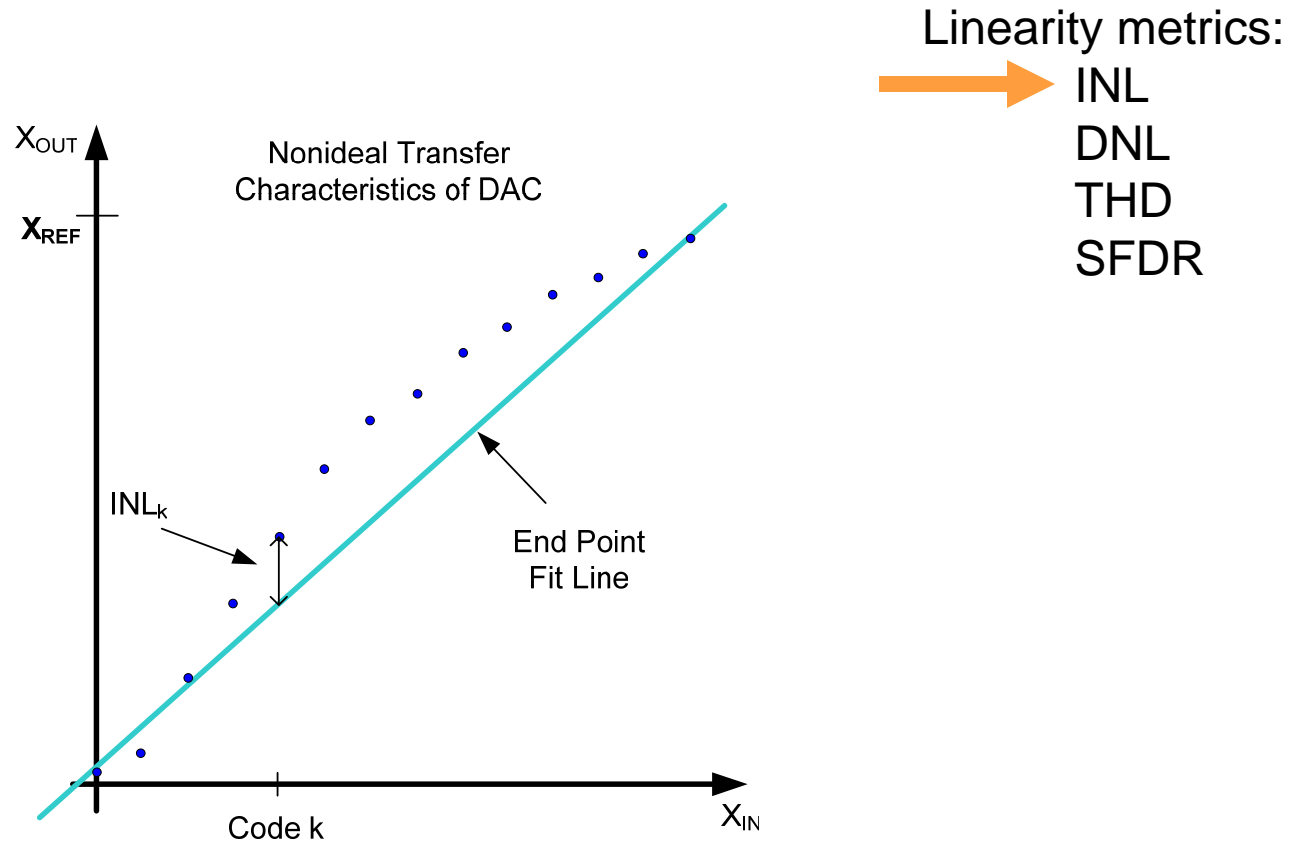
## Integral Nonlinearity (INL)

Measure of worst-case deviation from linear

Define the INL at any input code k by:

$$INL_k = X_{OUT}(k) - X_{FIT}(k)$$

# Integral Nonlinearity (INL)



Define the INL by:

$$INL = \max_{1 \leq k \leq N} \{ |INL_k| \}$$



# Integral Nonlinearity (INL)

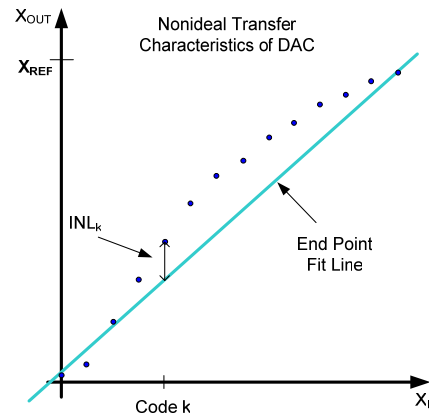
$$INL = \max_{1 \leq k \leq N} \{ |INL_k| \}$$

Often expressed in LSB:

$$INL_{LSB} = \frac{INL}{X_{LSBF}}$$

where

$$X_{LSBF} = \frac{X_{OUT}(N) - X_{OUT}(1)}{N-1}$$



Linearity metrics:



INL  
DNL  
THD  
SFDR

# Integral Nonlinearity (INL)

$$INL = \max_{1 \leq k \leq N} \{ |INL_k| \}$$

Often expressed in LSB:

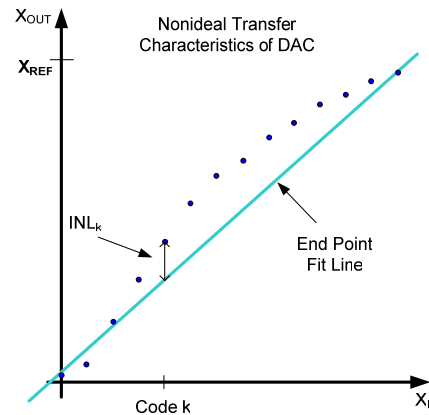
$$INL_{LSB} = \frac{INL}{X_{LSBF}}$$

where

$$X_{LSBF} = \frac{X_{OUT}(N) - X_{OUT}(1)}{N-1}$$

but

$$X_{LSBF} = \frac{X_{OUT}(N) - X_{OUT}(1)}{N-1} \cong \frac{X_{REF}}{2^n}$$



Linearity metrics:



INL  
DNL  
THD  
SFDR



$$INL_{LSB} \cong 2^n \frac{INL}{X_{REF}}$$

# Integral Nonlinearity (INL)

$$\text{INL} = \max_{1 \leq k \leq N} \{ |\text{INL}_k| \}$$

$$\text{INL}_{\text{LSB}} \cong 2^n \frac{\text{INL}}{X_{\text{REF}}}$$

What is the ideal INL?

$$\text{INL}_{\text{IDEAL}} = 0_{\text{LSB}}$$

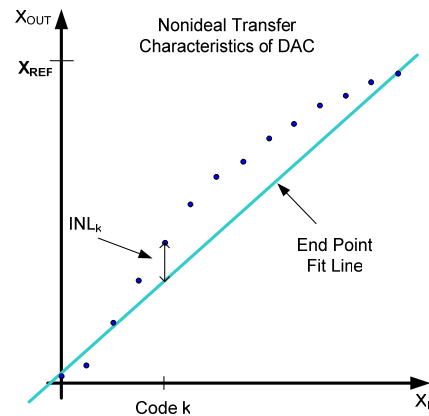
What is an acceptable INL?

If  $\text{INL} < 0.5\text{LSB}$ , it is generally considered acceptable

This would be the quantization error for an n-bit ADC

What is the INL of a DAC?

Varies from part to part, often close to  $0.5\text{LSB}$ , occasionally better, but often worse - Given in Data Sheet

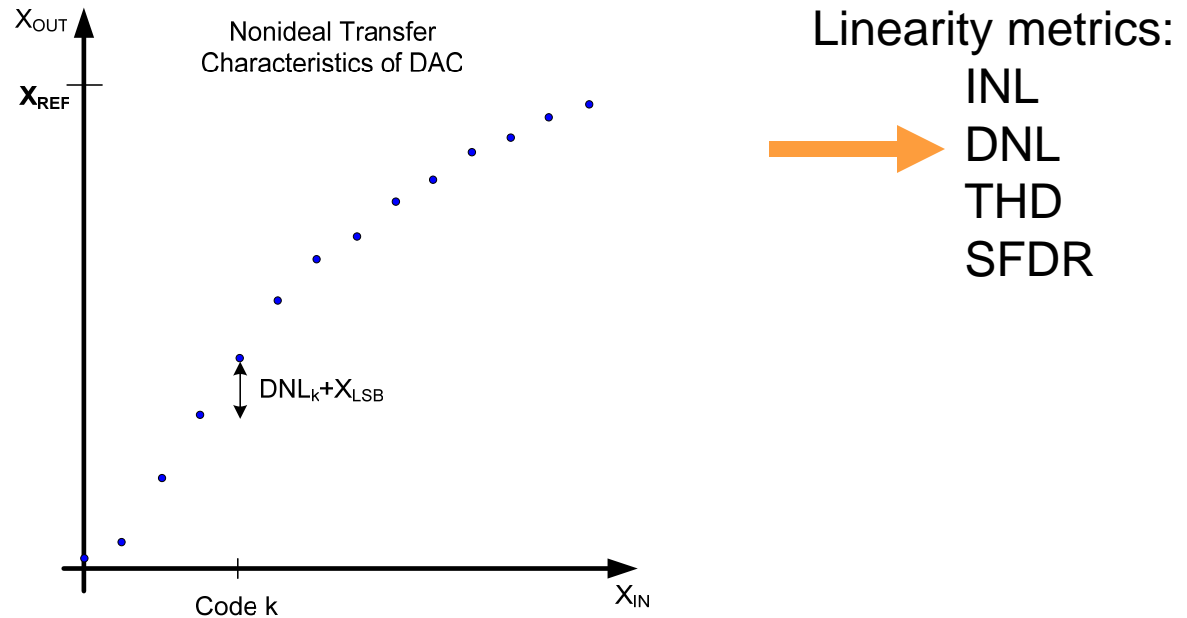


Linearity metrics:



INL  
DNL  
THD  
SFDR

# Characterization of Nonlinearities



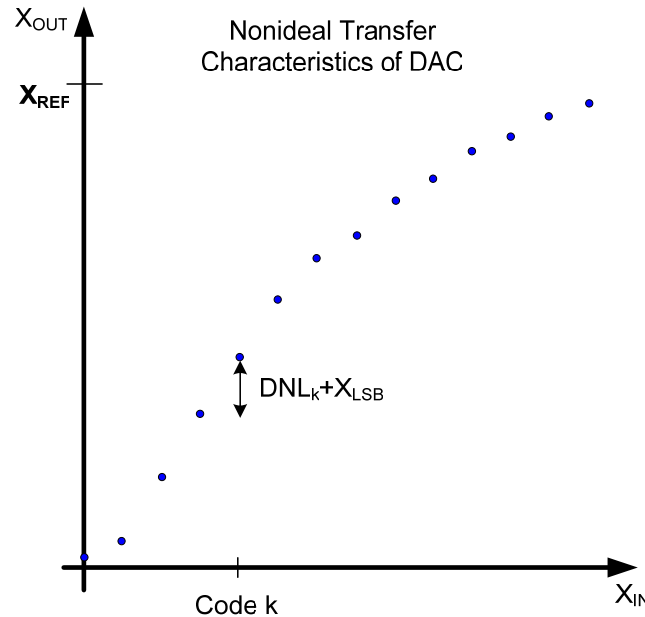
## Differential Nonlinearity (DNL)

Measure of worst-case resolving capabilities

Define the DNL at any input code  $k$  by:

$$DNL_k = X_{OUT}(k) - X_{OUT}(k-1) - X_{LSBF} \cong X_{OUT}(k) - X_{OUT}(k-1) - X_{LSB}$$

# Differential Nonlinearity (DNL)



Linearity metrics:

INL

DNL

THD

SFDR

$$DNL_k \cong X_{OUT}(k) - X_{OUT}(k-1) - X_{LSB}$$

$$DNL = \max_{1 < k \leq N} \{ |DNL_k| \}$$

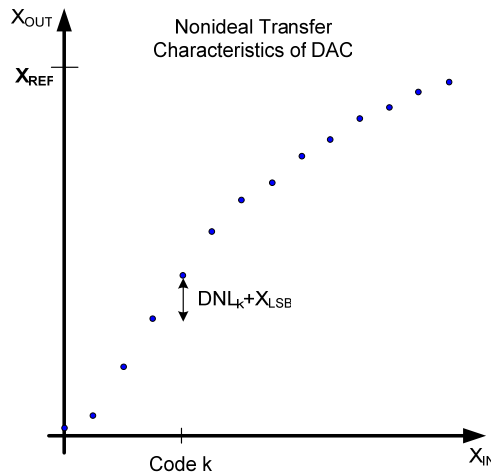
Often expressed in LSB

$$DNL_{LSB} = \frac{DNL}{X_{LSBF}}$$



$$DNL_{LSB} \cong 2^n \frac{DNL}{X_{REF}}$$

# Differential Nonlinearity (DNL)



Linearity metrics:

INL

DNL

THD

SFDR

$$DNL = \max_{1 < k \leq N} \left\{ \left| DNL_k \right| \right\}$$

$$DNL_{LSB} \cong 2^n \frac{DNL}{X_{REF}}$$

What is the IDEAL DNL of a DAC?

$$\text{Ideal DNL} = 0_{LSB}$$

What is an acceptable DNL of a DAC?

If  $DNL < 0.5LSB$ , it is generally considered acceptable

What is the INL of a DAC?

Varies from part to part, often close to  $0.5LSB$ , occasionally better, but often worse - Given in Data Sheet

**End of Lecture 42**